Development of new hybrid geoid model for Japan, "GSIGEO2011"

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Abstract

We have developed a new hybrid geoid model of Japan, "GSIGEO2011", which was made publicly available on April 1, 2014. The model was created in such a way that a high-resolution gravimetric geoid model for Japan, "JGEOID2008" was fitted to GNSS/leveling geoid undulations at 971 sites by the Least-Squares Collocation method. The model reproduces geoid heights at the GNSS/leveling sites with the consistency of a standard deviation of 1.8 cm. By utilizing the model to convert GNSS-derived three-dimensional positions to orthometric heights, GNSS survey can determine orthometric heights at the same precision as third-order leveling surveys. Orthometric height determination by GNSS surveying has been applicable in public surveys as alternative to third-order leveling in Japan since April 1 2014.

1. Introduction

The geoid is one of the equipotential surfaces of the Earth's gravity field, defined as the one that best fits global mean sea-level. The zero-height reference of the Japanese vertical datum is conventionally defined as the mean sea-level of Tokyo Bay, which is assumed to be the geoid.

Orthometric heights, defined as vertical distances from the geoid, are essential information in our daily life, because water naturally flows from higher to lower ground controlled by local orthometric height difference.

Fig. 1 shows the relationship between different height systems: orthometric height, ellipsoidal height and geoid height. The height of the geoid from the adopted reference ellipsoid is called the geoid height. The ellipsoidal-normal distance from the ellipsoid is the ellipsoidal height that can be measured by GNSS surveying. As Fig. 1 tells us, the orthometric height can be obtained by subtracting the geoid height from the corresponding ellipsoidal height. Therefore, we can determine orthometric heights anywhere by GNSS surveying if geoid heights are known everywhere.

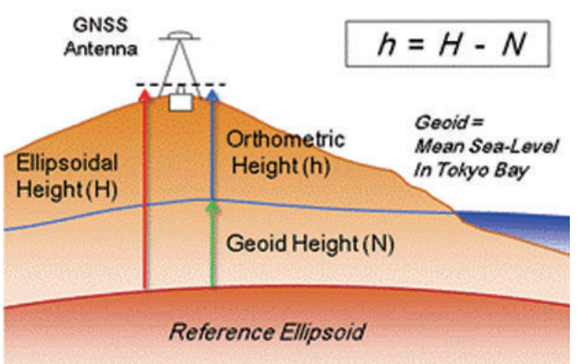

Fig. 1 Relationship between orthometric height, ellipsoidal height and geoid height.

The Geospatial Information Authority of Japan (GSI) developed its first Japanese hybrid geoid model, "GSIGEO2000" by combining a gravimetric geoid model for Japan with nationwide data of GNSS/leveling geoid undulations (Kuroishi et al., 2002), and made it publicly available on April 1 2002. The model has enabled us to determine orthometric heights directly from GNSS surveying. The accuracy of the model was evaluated at around 4cm in terms of standard deviation, which is precise enough to determine the orthometric heights of triangulation control points by GNSS survey. Accordingly, the model has been widely utilized in public surveys for orthometric height determination of triangulation control points in Japan.

Public demand for widening the application of GNSS observation to orthometric height determination with higher precision in order to make height surveys more efficient has grown. For such applications, the

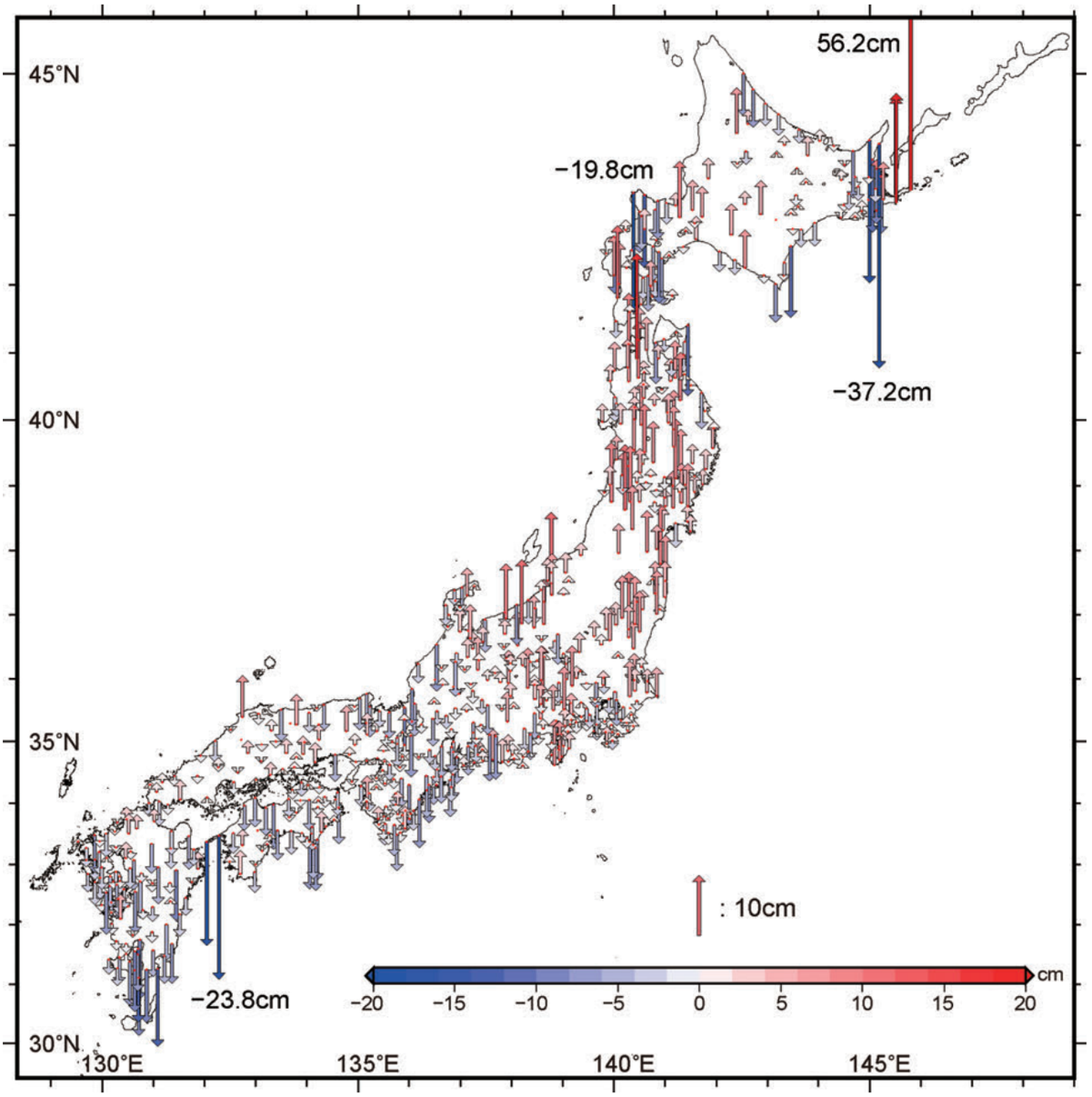

Fig. 2 Reproduction errors of GSIGEO2000 from Hokkaido to Kyushu at GNSS/leveling geoid undulation points used as input data in modeling.

hybrid geoid model is not satisfactorily precise. Fig. 2 shows the reproduction errors of GSIGEO2000 at the geoid undulation points used as input data in modeling. As you see, the error exceeds 10cm in some areas, mostly in peninsulas. Therefore, in order to meet the demand for accuracy, we need to develop a more accurate geoid model for the country.

In our new model development, we set up our goal to enable the combination of a geoid model with the GNSS survey to yield an orthometric height determination with the same level of precision as a third-order leveling survey. The model should have an accuracy of 2cm in terms of standard deviation.

Kuroishi (2009) developed the latest high-resolution gravimetric geoid model for Japan, JGEOID2008, on a grid of 1 by 1.5 minutes, which can be used as a foundational model because of its major improvement in precision. The GSI has been

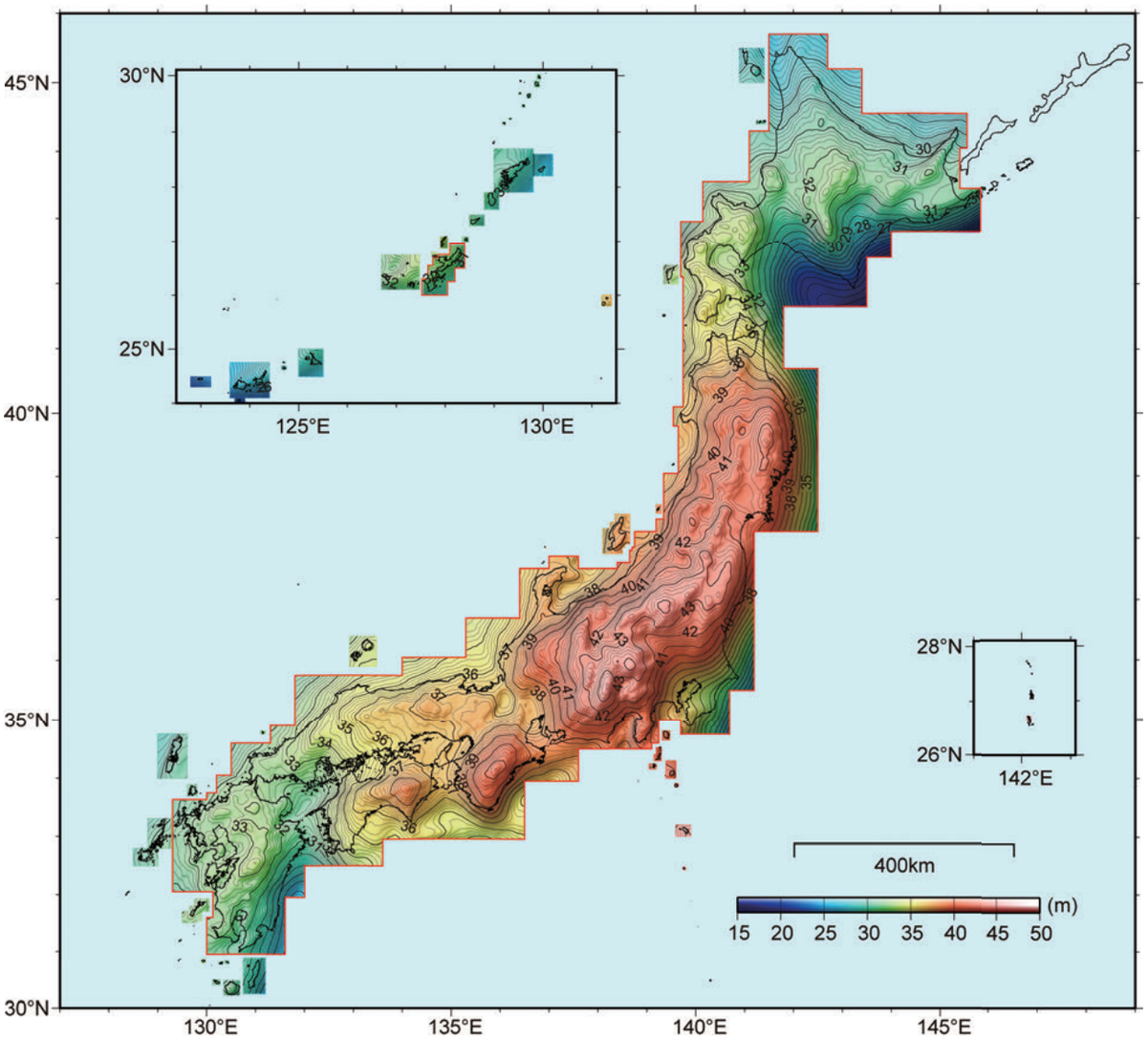

Fig. 3 New hybrid geoid model for Japan, GSIGEO2011. Counter interval is 20cm and numerals in meters. Areas enclosed with the red lines indicate the effective areas of the model.

conducting GNSS/leveling survey nation-wide at fairly homogeneously distributed sites with improved accuracy. Accordingly, we have developed a new hybrid geoid model for Japan, GSIGEO2011 (Fig. 3) from a combination of JGEOID2008 and newly-acquired GNSS/leveling undulations at 971 sites, which cover the main islands of Japan from Hokkaido to Kyushu at an average spacing of 25 km. The new model, which covers all Japanese islands except isolated islands, reproduces geoid undulations at GNSS/leveling geoid undulation points with a standard deviation of 2 cm, and is applicable to orthometric height determination directly from GNSS surveying for replacement of third-order leveling.

The GSI made the new model publicly available on April 1 2014. With this model, orthometric height determination by GNSS surveying has been applicable in public surveys as alternative to third-order leveling in such areas that are covered by the model.

2. Strategy of Model Development

The basic idea of the model development is that the high-resolution gravimetric geoid model is fitted to

the observed geoid undulation data by modeling a smooth correction model from discrepancies of geoid heights between the two. A schematic flow of the strategy is shown in Fig. 4, which resembles that of GSIGEO2000 development (Kuroishi et al, 2001).

First, we calculate geoid-height deviations of JGEOID2008 from the observed GNSS/leveling data at all sites. The deviations are decomposed into two components, namely ramp and residual geoid height deviations. The regional gravimetric geoid model may contain long and/or medium wavelength errors, whereas GNSS/leveling geoid undulations may include some bias, due to the possible inherent offset of the national vertical datum from the global geoid, and systematic cumulative errors associated with leveling surveys over long distances. As a result, the derived geoid-height deviations may contain some systematic spatial features.

In our present case, the deviations do exhibit a systematic pattern of long wavelength nature, as shown in the left hand side of Fig. 5. At a later stage of geoid correction modeling by the Least Squares Collocation method (LSC), such kinds of long wavelength features could deteriorate the accuracy of correction modeling, especially at short wavelengths. To reduce such effects, we estimate such a systematic pattern as ramp (a tilted planar model) optimally in a least squares sense and remove

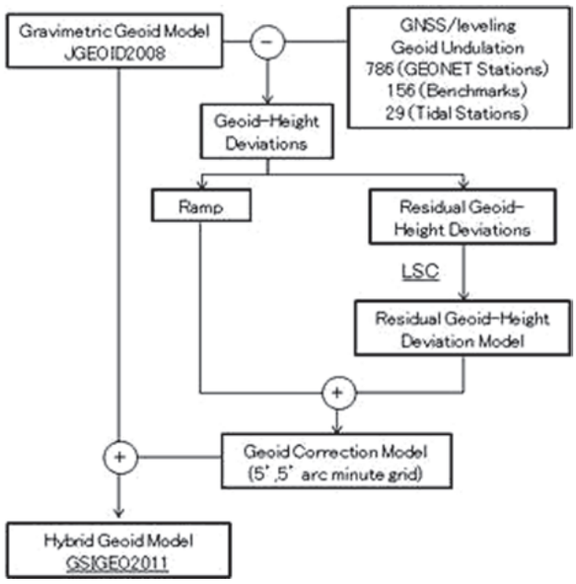

Fig. 4 Schematic flow of model development. The number of input geoid undulation data is for modeling of eastern Japan.

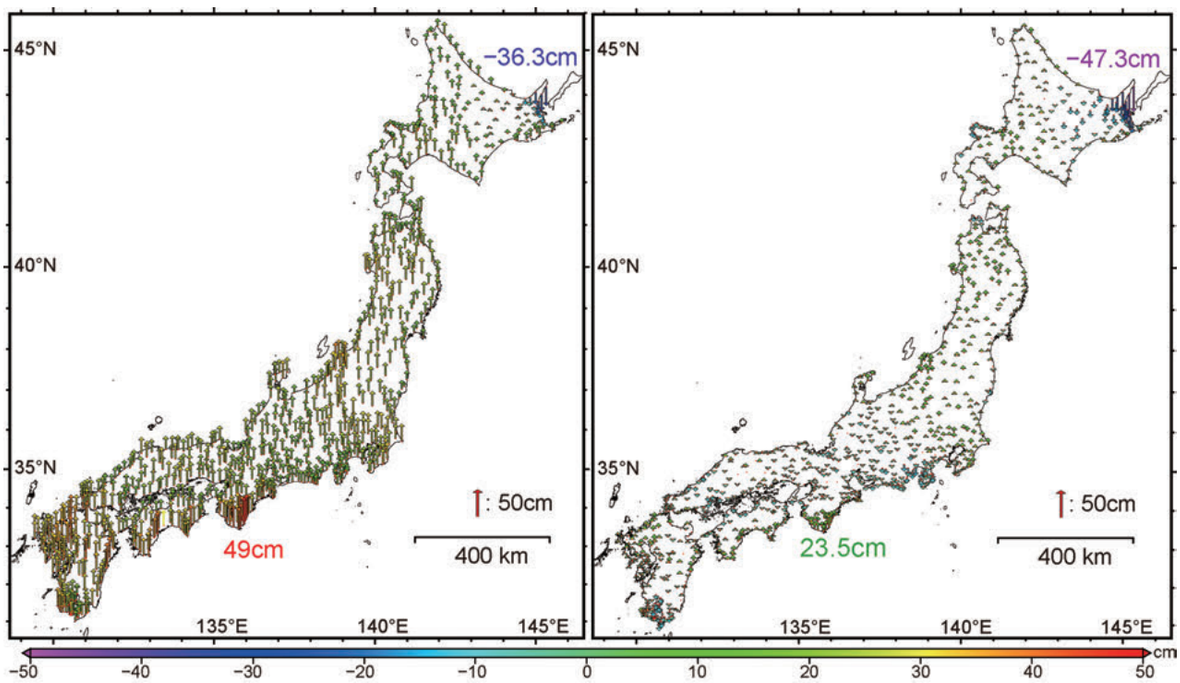

Fig. 5 Geoid-height deviations of JGEOID2008 from GNSS/leveling geoid undulations (left) and residual geoid-height deviations after removal of systematic ramp estimated (right).

the ramp component estimated from the geoid-height deviations beforehand with correction modeling. Residual geoid-height deviations thus processed are shown on the right hand side of Fig. 5 and used for modeling a geoid correction surface.

LSC is applied to the residual deviations for modeling a correction surface. Residual deviation data is distributed rather homogeneously and dominates variations over short wavelengths. Therefore, we can make the spatial resolution of the correction surface higher than that used in the development of the previous model, GSIGEO2000; we set a grid of 5 by minutes. In the LSC process, we need to determine an analytical covariance function, which should optimally match empirical covariance functions. We assume multiple Gaussian functions (exponential functions of different amplitudes with different correlation lengths).

A final hybrid geoid model is created by adding the estimated correction surface and the ramp component back to JGEOID2008.

3. GNSS/leveling Derived Geoid Undulation Data

The GSI has been carrying out GNSS/leveling survey nationwide with improved precision and geoid undulations were newly obtained at 971 sites for geoid model development. Fig. 6 shows the distribution of those sites. Most of the data were obtained at 786 sites out of more than 1,300 of GNSS continuously operating reference stations operated by the GSI, named the GNSS Earth Observation Network System (GEONET). Second-order leveling surveys were conducted from the adjacent first-order leveling routes to each of those stations, yielding orthometric heights. The GSI also established and has been operating GNSS continuous observation stations

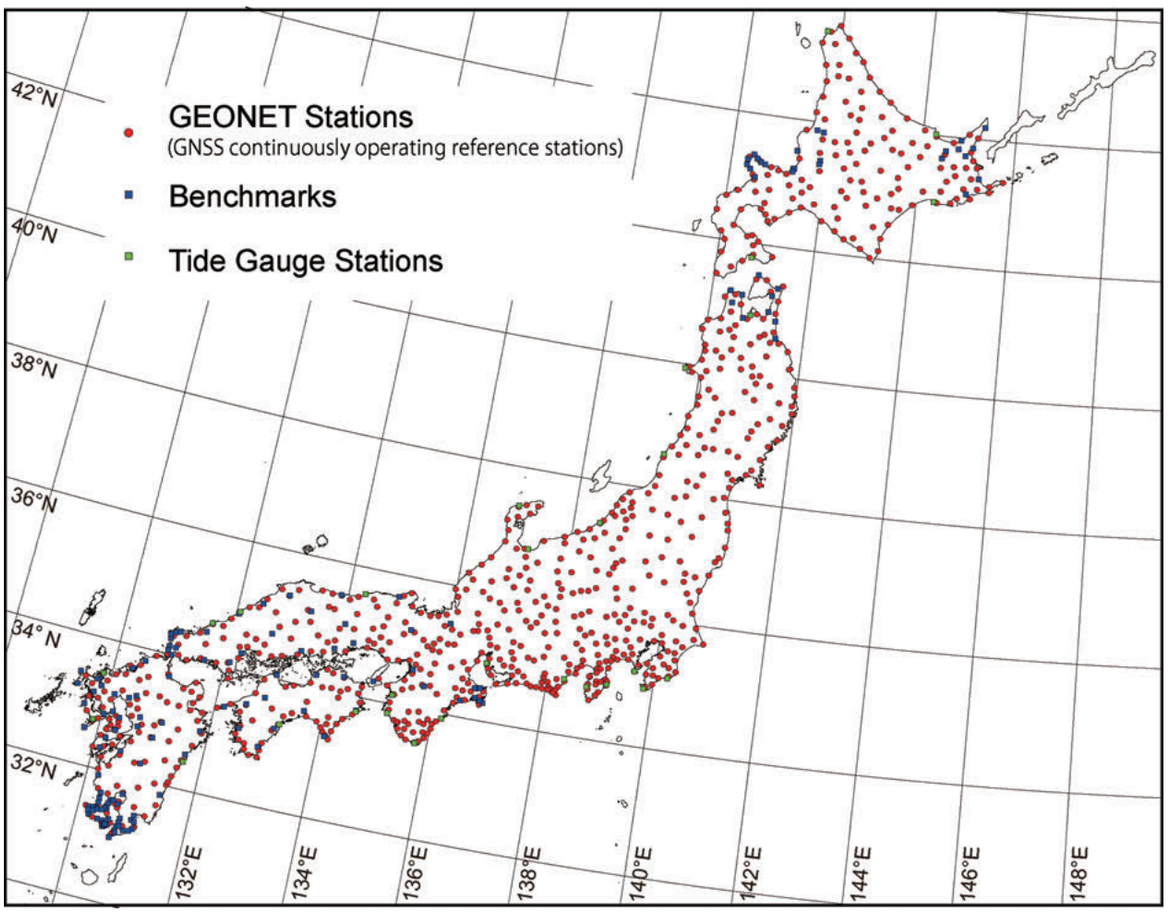

Fig.6 Distribution of input GNSS/leveling data from Hokkaido to Kyushu.

and benchmarks at tide gauge stations and first-order leveling surveys were conducted from the adjacent first-order leveling route to the benchmarks. Geoid undulations at 29 tide gauge stations in total are used. In addition, a campaign type of GNSS observation was made at some benchmarks to enhance geoid undulation data. Eventually geoid undulations were obtained at 156 benchmarks for modeling.

4. Development of Hybrid Geoid Model, GSIGEO2011

Initially, the GSI planned to develop a hybrid geoid model for the entire nation and GNSS/leveling survey for geoid undulation measurements had been conducted in a well-planned manner throughout Japan. However, the 2011 Off Tohoku Earthquake of Magnitude 9.0 on March 11, 2011 brought about a huge amount of crustal deformation across eastern Japan, resulting in some of the already-acquired geoid undulations needing to be re-observed if geoid heights were significantly changed. On the other hand, demands for enhancing GNSS survey applications to precise orthometric height determination have been growing, and the GSI decided to develop and make publicly available an improved geoid model with required precision as early ahead as possible, for the areas where geoid height changes associated with the earthquake are not significant, namely for western Japan (Fig. 7).

4. 1 Geoid Model for the Western Part of Japan

GNSS/leveling geoid undulations are used for modeling the western part (Kyushu, Chugoku and Shikoku Islands): 356 GEONET stations, 16 tide gauge stations and 121 benchmarks. In order to avoid edge effects (errors) in modeling the correction surface, the area of computation and data distribution includes not only Kyushu, Chugoku and Shikoku, but also the Kinki district and a part of the Chubu district, namely Shizuoka, Aichi and Mie prefectures. The targeted areas were extracted from the resulting hybrid geoid model and the thus-obtained model, called GSIGEO2011, was made publicly available on April 1, 2013.

To determine an analytical covariance function for LSC, an empirical covariance function of the residual geoid undulation deviations of JGEOID2008 from the geoid undulation data was computed at bins every 5 arc-minutes of ellipsoidal baseline lengths. For example, the empirical covariance at 0 minutes is calculated from all pairs of baseline length from 0 to 2.5 minutes, and that at 5 minutes from 2.5 to 7.5 minutes. A multiple Gaussian function model, as used in Roman et al. (2010), is assumed here for the analytical covariance function. Fig. 8 shows the results of empirical and analytical covariance functions for the western part of Japan and for the main islands of Japan (hereinafter called the whole area of Japan) to be discussed in the following section.

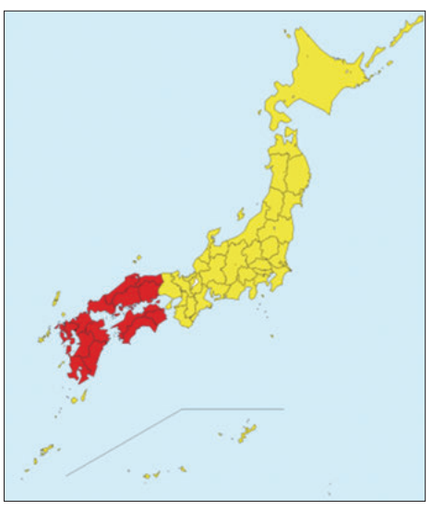

Fig. 7 Western part of Japan in red.

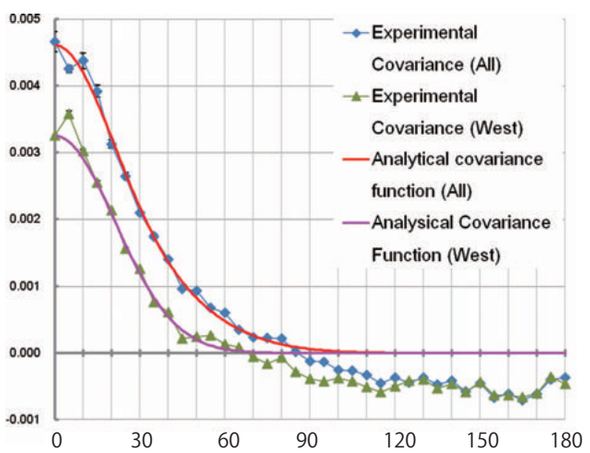

Fig. 8 Empirical and analytical covariance functions for modeling western part and whole area of Japan.

Similar patterns are commonly observed in both cases; the covariance functions diminish rather quickly and show almost no correlation over some 60 minutes as we see. The parameters of the analytical covariance functions are given in Table 1. The amplitude is entirely smaller in the western part case than in the whole area case. This might imply a difference in amplitudes of geoid undulations over short wavelengths between the two areas.

Table 1 Parameters of the analytical covariance functions determined for the western part and whole area of Japan.

Whole area	Correlation Length (deg.)	26	47
	Amplitude (m^2)	0.049	0.047
Western part	Correlation Length (deg.)	30	
	Amplitude (m^2)	0.057	

Fig. 9 shows reproduction errors of GSIGEO2011 for the western part at GNSS/leveling geoid undulation points used as input data in modeling. Table 2 compiles statistics of the internal errors and indicates that the model reproduces geoid heights at the GNSS/leveling undulation points with the consistency of a standard deviation of 2.0 cm. The errors range from -6.2 cm to 6.0 cm with an average of 0.0 cm.

To evaluate the model semi-externally, we also tried a 2-fold cross validation and leave-one-out cross validation (LOOCV). In the 2-fold validation, geoid undulation data are first divided into two groups with the same number of data and with spatial distribution as equally as possible, and a hybrid geoid model is computed with the same strategy as that used for GSIGEO2011 by using one of the groups and is compared to the other group as independent data. In our present case, we sorted the data by latitude and picked them up alternately into

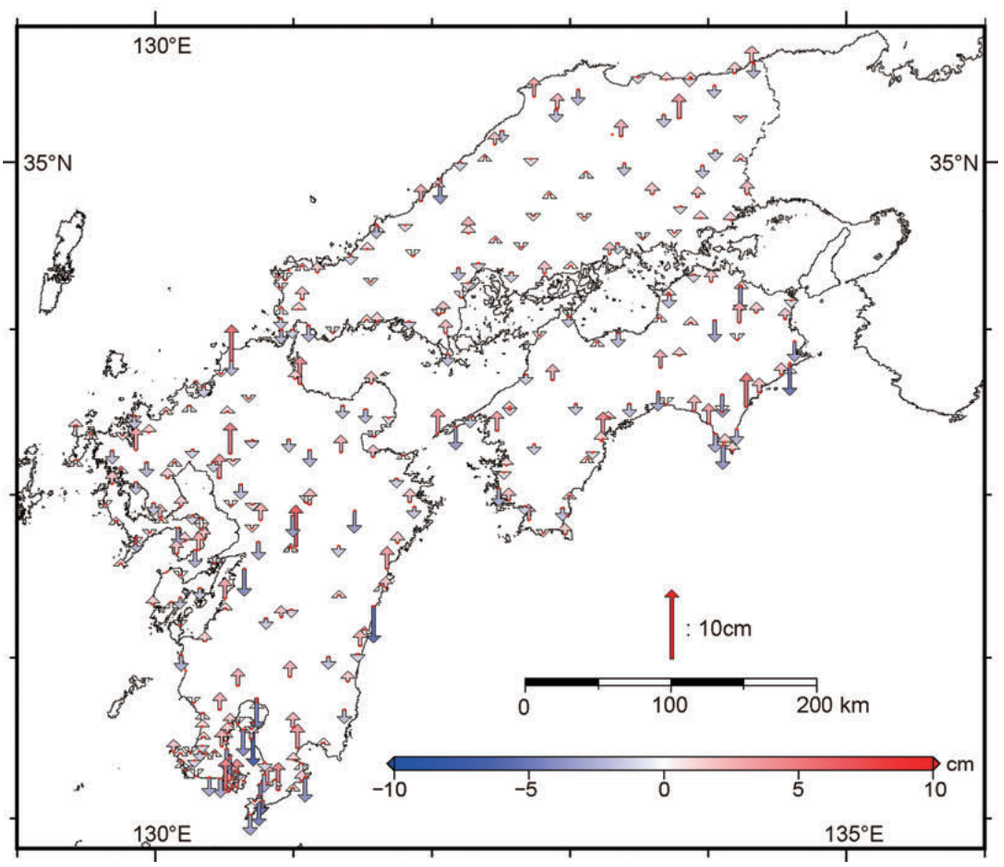

Fig. 9 Reproduction errors of GSIGEO2011 at GNSS/leveling geoid undulation points used as input data in modeling over western part of Japan.

one group and another. In LOOCV, a hybrid geoid model is computed without any of the geoid undulation data and the resulting model is compared to the ignored data as independent data in terms of geoid height: the process is repeated until all data are used for external comparison, yielding error statistics. The results for both validations are also given in Table 2 and show slightly larger errors than those of the direct comparison.

Table 2 Statistics of geoid height errors of GSIGEO2011 for the western part of Japan.

Validation Method	Direct	2-fold CV		LOOCV
Input data number	All	A	B	487
Data for comp.	All	B	A	1
No. for comp.	488	244	244	488
Average (cm)	0.0	0.0	0.0	0.0
Standard Dev. (cm)	2.0	2.2	2.3	2.6
Max. Error. (cm)	6.0	7.5	7.6	8.9
Max. Error (cm)	-6.2	-7.8	-7.0	-7.5

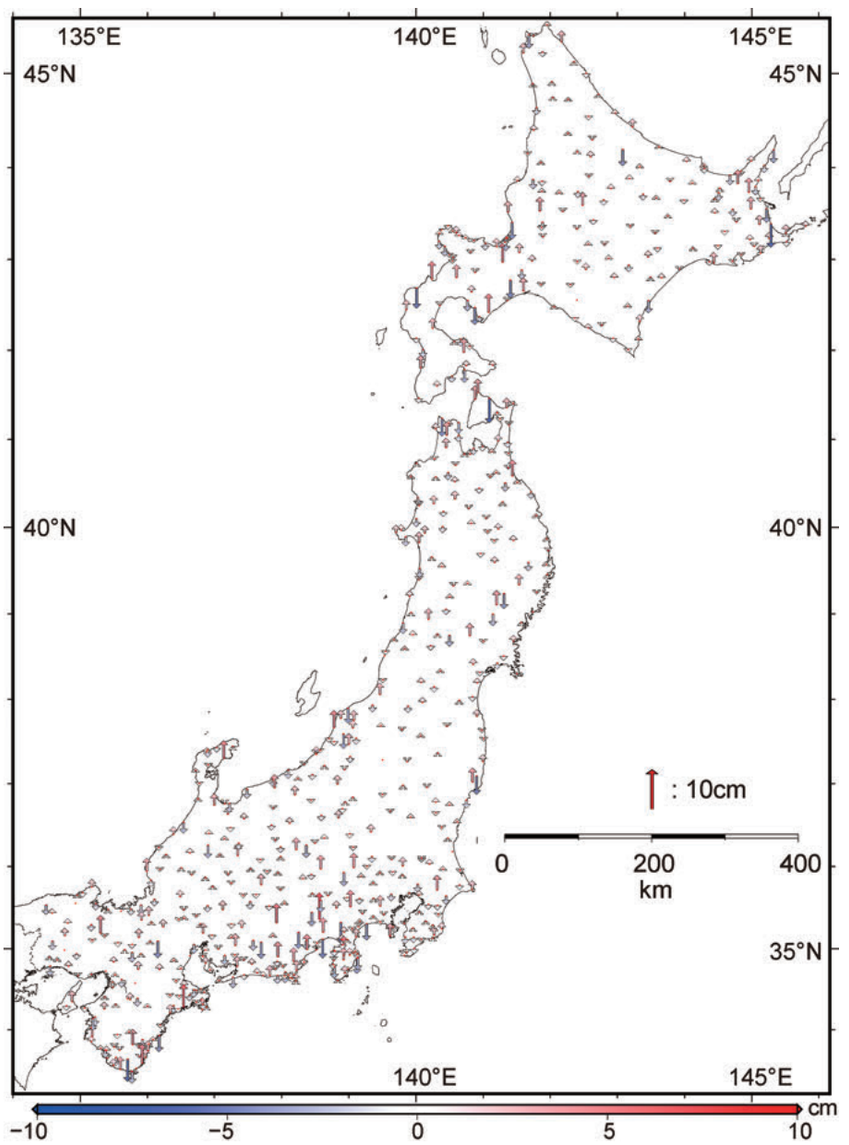

Fig. 10 Reproduction errors of GSIGEO2011 at GNSS/leveling geoid undulation points used as input data in modeling over eastern part of Japan.

4.2 Geoid Model for Eastern Part of Japan

A hybrid geoid model for the eastern part of Japan was newly developed after the completion of re-observation of GNSS/leveling in the areas where crustal deformation associated with the 2011 Off the Pacific coast of Tohoku Earthquake affected significantly in geoid undulations. First, a hybrid geoid model was computed with geoid undulation data covering the whole area of Japan, namely including the western part of Japan. Although the analytical covariance functions determined are different from each other as seen in the previous section, no substantial differences are found in the western part of Japan between the new model for the whole area and the previously released model for the western part of Japan. Therefore, we replaced the new model by the previously released geoid model for the western part of Japan over the area concerned and obtained the final geoid model for the whole area of Japan, called GSIGEO2011 ver.1. By this handling, we wouldn't cause any trouble (confusion) to users of the previously-release model, GSIGE2011 and the model, GSIGEO2011 ver.1, was released on April 1 2014.

Next, we evaluate the model over the eastern part of Japan. In that area the number of GNSS/leveling geoid undulation data used were 971 in total: 786 GEONET stations, 29 tide gauge stations and 156 benchmarks.

Fig. 10 shows reproduction errors of GSIGEO2011 at GNSS/leveling geoid undulation points used as input data in modeling over the area concerned. The model reproduces geoid heights at the GNSS/leveling undulation points with the consistency of a standard deviation of 1.8 cm. The errors range from -6.2 cm to 6.7 cm with average of 0.0 cm.

The same semi-external evaluation as applied in the previous section is also made and resulting statistics of errors are compiled in Table 3. The increase rates of standard deviations and ranges with respect to the direct comparison become larger than those in the previous section for the western part of Japan. This might reflect the difference in spatial density of geoid undulation data: the special coverage of GNSS/leveling geoid undulations is less dense in the eastern part of Japan, notably in the Hokkaido and Tohoku districts (Fig. 6), where distances between the neighboring data sometimes reach 50 km.

The methods employed for semi-external evaluation could degrade the resulting model through the removal of data at comparison location. The enlargement of the increase rates in the eastern part might indicate that the spatial density of the geoid undulation data is not satisfactory to reveal the geoid undulation at short wavelengths over that area. If this is the case, the geoid model obtained would be less precise locally in some locations where the data distribute less densely.

Table 3 Statistics of geoid height errors of GSIGEO2011 for the eastern part of Japan.

Validation Method	Direct	2-fold CV		LOOCV
Input data number	All	A	B	970
Data for comp.	All	B	A	1
No. for comp.	971	486	484	971
Average (cm)	0.0	0.0	0.0	0.0
Standard Dev. (cm)	1.8	2.3	2.4	2.8
Max. Error (cm)	6.7	9.8	10.8	9.7
Min. Error (cm)	-6.2	-8.4	-11.3	-11.7

4.3 Internal Consistency of the Final Geoid Model

We finally evaluate the geoid model, GSIGEO2011 internally for the whole area. Fig. 11 shows reproduction errors of GSIGEO2011 at GNSS/leveling geoid undulation points used as input data in modeling. The model reproduces geoid heights at the GNSS/leveling undulation points with the consistency of a standard deviation of 1.8 cm. The errors range from -6.2 cm to 8.2 cm with average of 0.0 cm, indicating that GSIGEO2011 is consistent with GNSS/leveling geoid heights to 2 cm in magnitude at one-sigma level of confidence and that it achieves our target in consistency. Then, GSIGEO2011 is applicable to orthometric height determination directly from GNSS surveying for replacement of third-order leveling in such areas that are covered by the model.

5. Conclusions

We have developed a new hybrid geoid model for Japan, GSIGEO2011, on a grid of 1 by 1.5 minutes. The model reproduces geoid heights at the GNSS/leveling sites with the consistency of a standard deviation of 1.8 cm, and is applicable to orthometric height determination

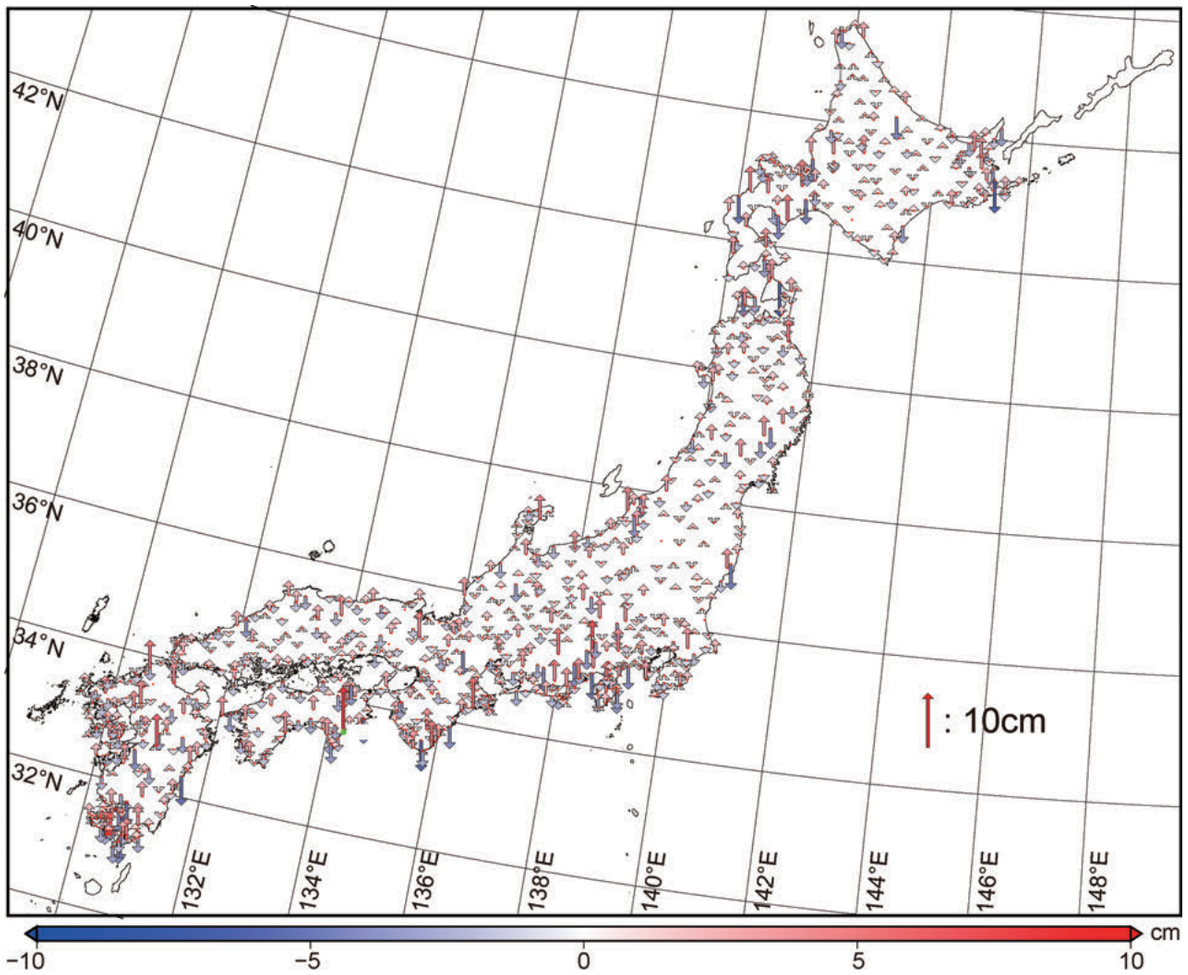

Fig. 11 Reproduction errors of GSIGEO2011 from Hokkaido to Kyushu at GNSS/leveling geoid undulation points used as input data in modeling over the whole area.

directly from GNSS surveying for replacement of third-order leveling. The GSI made the model publicly available on April 1 2014. With this model, orthometric height determination by GNSS surveying has been applicable in public surveys as an alternative to third-order leveling in such areas which are covered by the model. For further improvement, additional data are essential in the areas of poor spatial coverage of geoid undulations data.

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